Proto Math 339 Final April 2009

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1 To Sue or Settle

I burned my mouth on a cup of hot tea at a McDonald's the other day, when my litigious eye spied that they had forgoten to put the hot sleeve on the cup, so there was no warning that the contents were hot. I'm planning to sue them for 100 million dollars. I figure that if the case goes to trial I have a $\frac{3}{4}$ chance of winning the 100 million, if I lose I get nothing. McDonald's thinks that the case goes to trial I only have a $\frac{1}{2}$ chance of winning.

I'm representing myself since I'm a grad student with lots of free time (or at least I will be once this exam is marked) so the trial costs me nothing, but McDonald's is going to spend a ton on defence to the tune of 28 million dollars.

Suppose McDonald's can offer me an out of court settlement before the case goes to trial. Assume that the settlement amount is an integer number of millions.

The game then plays out as follows. First McDonald's makes a settlement offer of x million. If I accept the offer and the game is over with the payoffs of $-x$ million for McDonald's and x million for me. If I reject the offer the case goes to trial, and my payoff is what I expect from the outcome of the trial, and and McDonald's payoff is what they expect to get out of the trial minus the cost of defence.

1. Use backward induction to figure out the subgame perfect Nash equilibrium of the game. Do we settle or not? If we do what is the settlement offer that is expected?

2. Suppose the court system wants to encourage more settlements and so they institute a new rule where by if the person doing the suing rejects a settlement offer and looses the trial then they have to pay for the defence. Use Backward induction to figure out the subgame perfect Nash equilibrium of the game under this rule. Do we settle or not? If we do what is the settlement offer that is expected?

3. Did this new rule encourage settlement or not? If it worked explain why, if it failed explain why.

2 Who Called The Cops

D denizens in a neighbourhood all witness a crime going down out on the street from their apartment windows. Everyone in the hood has to simultaneously and independently decide whether or not to call the police. If a denizen calls the police there is a good chance they will have to spend a fair amount of time talking with the cops later, and be called in as a witness in the ensuing court trial, and hence they pay a cost c in time if they call. If at least one person calls the police then the police arrive in time make some justice and everyone in the hood loves justice so everyone get payoff u if they didn't call and $u - c$ if they did call. If no one calls no justice is done, and so everyone in the hood has payoff 0. Assume $0 < c < u$.

1. For $D = 1$ find all the Nash equilibria both pure and mixed.
2. For $D = 2$ find all the Nash equilibria both pure and mixed.
3. Find all the Nash equilibria, both pure and mixed, for arbitrary D.
4. Are people more or less likely to call in a large neighbourhood. What happens to the probability of help being called for as the number of denizens in the neighbourhood increases.

3 Matrix Game Acrobatics

Consider the following matrix game between player S and player T:

	t_1	t_2
s_1	x, y	2,2
s_2	3,1	3,4

1. What must be true of x and y for (s_1, t_1) to be a strict Nash equilibrium?
2. What must be true of x and y for (s_1, t_1) to be a strict Nash equilibrium?
3. What must be true of x and y for there to be a Mixed Strategy Nash Equilibrium where player S plays s_1 with probability $p^* = \frac{3}{4}$ and player T plays t_1 with probability $q^* = \frac{1}{3}$?
4. What are the expected payoffs of each player at the MSNE found in part 3.

4 Continuous Strategies

Two math geeks play a procrastinating game where they get a computer to generate two random payoff function, one for each of them. They then simultaneously play a real number between 1 and 5, and calculate the payoffs from their respective functions. If they both play the Nash equilibrium then they both eat an Oreo and play again, otherwise they have to switch to real homework.

Suppose the following to payoff functions were generated.

$$\pi_1(x, y) = -x^2 + 6x + y \tag{1}$$

$$\pi_2(x, y) = \frac{-y^2}{x} + y \tag{2}$$

Where player 1 plays x and player 2 plays y and $x, y \in [0, 5]$.

1. Find the players' best response functions, be sure to check boundary points.
2. Find the Nash by finding the intersection of the two players best response functions.

5 A Saddle Shaped Knife Edge

Suppose there is a population of ferns and a population of deer. The ferns can be one of two types, either tasty (T) or bitter (B), and the deer can be one of two types either a considerate muncher (C) or gorger (G). The considerate munchers, eat only small nibbles from the tasty ferns so that they can grow back and are careful to not step on tasty ferns, while crushing bitter ferns under hoof at every opportunity. Gorging deer eat wildly crushing all kinds of ferns underfoot without care, however they will eat tasty ferns right down to the root, but once they have one bite of a bitter fern they spit it out and try to find some tasty ferns.

	T	B
C	3,2	0,0
G	2,1	1,3

1. Find the mixed strategy Nash equilibrium (MSNE) of this game. (5 marks)
2. Write out the replicator equations for p and q where p is the proportion of deer playing considerate, and q is the proportion of ferns playing tasty. Note that $1 - p$ deer play gorging and $1 - q$ of the ferns play bitter. (5 marks)
3. Find the isoclines of the system determined by the replicator equations. (5 marks)
4. Use the intersections of the isoclines to determine the equilibria of the system, you should find five. (5 marks)
5. Calculate the Jacobian of the system. (5 marks)
6. Use the cheat sheet on the board to calculate the stability and type of the each equilibrium by evaluating the Jacobian at each equilibrium point. (5 marks)
7. Draw a picture of the isoclines and some arrows showing which way the trajectories go. (5 marks)
8. Tell a story about the dynamics of this population. (5 marks)

6 Working Backwards

Fill out the following matrix in such a way that it is an evolutionary game (two player symmetric) with a non-trivial MSNE (one in which each of the three strategies is played with non-zero probability) and have this MSNE also be an ESS.

	b_1	b_2	b_3
a_1	(,)	(,)	(,)
a_2	(,)	(,)	(,)
a_3	(,)	(,)	(,)